3.1. Derivacija

30.10. 2020.

Fotografija: Sean (Unsplash)

Motivacija 1: Problem trenutne brzine

Neka je

$$s(t) = \text{put koji promatrani automobil prijeđe do trenutka } t > 0.$$

Kako odrediti brzinu kojom se automobil kreće u trenutku $t_0 > 0$?

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Za svaki $t > 0$, $t \neq t_0$, srednja brzina automobila u vremenskom intervalu s rubovima t i t_0 iznosi

$$\frac{s(t) - s(t_0)}{t - t_0}.$$

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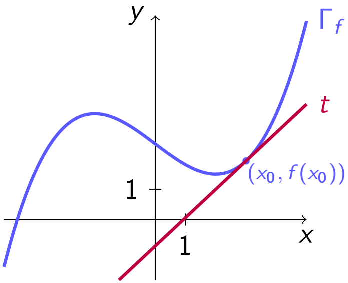
$$\frac{s(t) - s(t_0)}{t - t_0}.$$

$\rightsquigarrow$ Brzina automobila u trenutku t_0 iznosi

$$s'(t_0) := \lim_{t \to t_0} \frac{s(t) - s(t_0)}{t - t_0}.$$

Motivacija 2: Koeficijent smjera tangente na Γ_f

Kako odrediti koeficijent smjera tangente t na Γ_f povučene u točki $(x_0, f(x_0))$?



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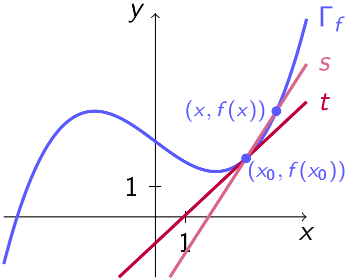

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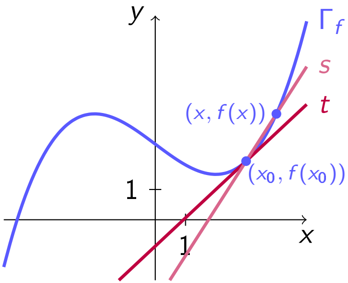

Za $x \neq x_0$, koeficijent smjera sekante s kroz točke $(x_0, f(x_0))$ i $(x, f(x))$ je

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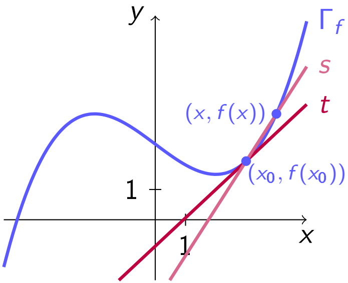

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$$\frac{f(x) - f(x_0)}{x - x_0}.$$

$\rightsquigarrow$ Koeficijent smjera tangente t je

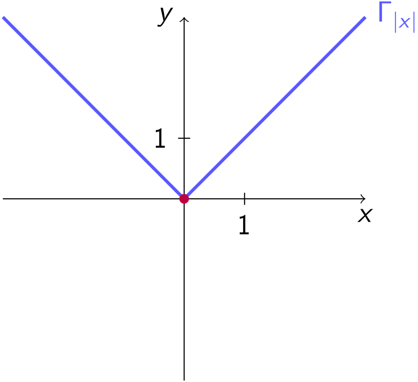
$$f'(x) := \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Nisu sve funkcije svuda derivabilne!

Primjer. Funkcija $|\cdot| : \mathbb{R} \to \mathbb{R}$,

$$|x| := \begin{cases} x, & \text{ako je } x \geq 0, \\ -x, & \text{ako je } x < 0, \end{cases}$$

nije derivabilna u 0.



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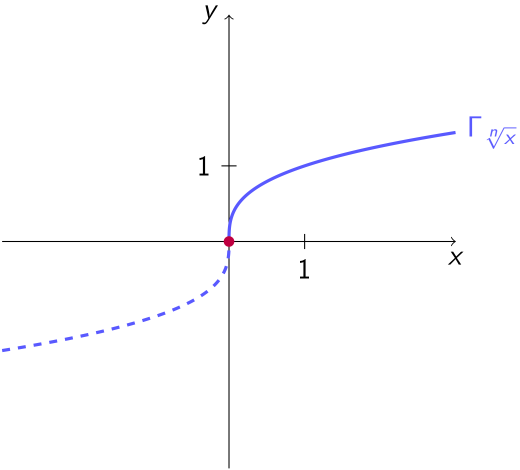
Primjer. Ni za koji prirodan broj $n \geq 2$ funkcija $\sqrt[n]{\cdot}$ nije derivabilna u 0.



Tablica derivacija

$$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$$

$$x' = 1$$

$$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$$

$$(\sin x)' = \cos x$$

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$$(a^x)' = a^x \ln a \quad (a > 0)$$

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Pravilo 1

Za funkcije $f_1, \dots, f_n : D \subseteq \mathbb{R} \to \mathbb{R}$ i $a_1, \dots, a_n \in \mathbb{R}$ vrijedi

$$(a_1 f_1 + a_2 f_2 + \ldots + a_n f_n)' = a_1 f_1' + a_2 f_2' + \ldots + a_n f_n',$$

kad god je desna strana definirana.

Zadatak 21(a)

Derivirajte funkciju

$$f(x) := x^5 - 4x^3 + 2x - 3.$$

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$$f(x) := x^5 - 4x^3 + 2x - 3.$$

Rješenje. Imamo

$$f'(x) = \left(x^5\right)' - 4\left(x^3\right)' + 2x' - 3'$$

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Rješenje. Imamo

$$f'(x) = \left(x^5\right)' - 4\left(x^3\right)' + 2x' - 3'$$
$$= 5x^4 - 4 \cdot 3x^2 + 2 \cdot 1 - 0$$

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$$f(x) := x^5 - 4x^3 + 2x - 3.$$

Rješenje. Imamo

$$\begin{aligned} f'(x) &= \left(x^5\right)' - 4\left(x^3\right)' + 2x' - 3' \\ &= 5x^4 - 4 \cdot 3x^2 + 2 \cdot 1 - 0 \\ &= 5x^4 - 12x^2 + 2. \end{aligned}$$

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Zadatak 21(b)

Neka su zadani $a, b \in \mathbb{R} \setminus \{0\}$. Derivirajte funkciju

$$f(x) := \frac{ax^6 + b}{\sqrt{a^2 + b^2}}.$$

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Rješenje. Imamo

$$f'(x) = \left(\frac{a}{\sqrt{a^2 + b^2}} x^6 + \frac{b}{\sqrt{a^2 + b^2}} \right)'$$

$$= \frac{a}{\sqrt{a^2 + b^2}} \left(x^6\right)' + \left(\frac{b}{\sqrt{a^2 + b^2}} \right)'$$

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$$= \frac{a}{\sqrt{a^2 + b^2}} \left(x^6\right)' + \left(\frac{b}{\sqrt{a^2 + b^2}}\right)'$$

$$= \frac{a}{\sqrt{a^2 + b^2}} \cdot 6x^5 + 0$$

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Rješenje. Imamo

$$\begin{aligned} f'(x) &= \left(\frac{a}{\sqrt{a^2 + b^2}} x^6 + \frac{b}{\sqrt{a^2 + b^2}} \right)' \\ &= \frac{a}{\sqrt{a^2 + b^2}} \left(x^6\right)' + \left(\frac{b}{\sqrt{a^2 + b^2}} \right)' \\ &= \frac{a}{\sqrt{a^2 + b^2}} \cdot 6x^5 + 0 \\ &= \frac{6ax^5}{\sqrt{a^2 + b^2}}. \end{aligned}$$

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Zadatak 21(c)

Derivirajte funkciju

$$f(x) := 3x^{\frac{2}{3}} - 2x^{\frac{5}{2}} + x^{-3}.$$

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$$f(x) := 3x^{\frac{2}{3}} - 2x^{\frac{5}{2}} + x^{-3}.$$

Rješenje. Imamo

$$f'(x) = 3 \cdot \frac{2}{3} x^{\frac{2}{3}-1} - 2 \cdot \frac{5}{2} x^{\frac{5}{2}-1} + (-3)x^{-3-1}$$

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$$= 2x^{-\frac{1}{3}} - 5x^{\frac{3}{2}} - 3x^{-4}.$$

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$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(d)

Derivirajte funkciju

$$f(x) := \frac{1}{\sqrt[3]{x^2}} - \frac{3}{x\sqrt[3]{x}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(d)

Derivirajte funkciju

$$f(x) := \frac{1}{\sqrt[3]{x^2}} - \frac{3}{x\sqrt[3]{x}}.$$

Rješenje. Imamo

$$f'(x) = \left(x^{-\frac{2}{3}} - 3x^{-\frac{4}{3}}\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(d)

Derivirajte funkciju

$$f(x) := \frac{1}{\sqrt[3]{x^2}} - \frac{3}{x\sqrt[3]{x}}.$$

Rješenje. Imamo

$$f'(x) = \left(x^{-\frac{2}{3}} - 3x^{-\frac{4}{3}}\right)'$$
$$= -\frac{2}{3}x^{-\frac{5}{3}} - 3\left(-\frac{4}{3}\right)x^{-\frac{7}{3}}$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(d)

Derivirajte funkciju

$$f(x) := \frac{1}{\sqrt[3]{x^2}} - \frac{3}{x\sqrt[3]{x}}.$$

Rješenje. Imamo

$$f'(x) = \left(x^{-\frac{2}{3}} - 3x^{-\frac{4}{3}}\right)' = -\frac{2}{3}x^{-\frac{5}{3}} - 3\left(-\frac{4}{3}\right)x^{-\frac{7}{3}} = -\frac{2}{3}x^{-\frac{5}{3}} + 4x^{-\frac{7}{3}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(e)

Derivirajte funkciju

$$f(x) := \operatorname{tg} x - \operatorname{ctg} x.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(e)

Derivirajte funkciju

$$f(x) := \operatorname{tg} x - \operatorname{ctg} x.$$

Rješenje. Imamo

$$f'(x) = \frac{1}{\cos^2 x} - \left(-\frac{1}{\sin^2 x}\right)$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(e)

Derivirajte funkciju

$$f(x) := \operatorname{tg} x - \operatorname{ctg} x.$$

Rješenje. Imamo

$$f'(x) = \frac{1}{\cos^2 x} - \left(-\frac{1}{\sin^2 x}\right) = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(f)

Derivirajte funkciju

$$f(x) := \operatorname{arctg} x + \operatorname{arcctg} x.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(f)

Derivirajte funkciju

$$f(x) := \operatorname{arctg} x + \operatorname{arcctg} x.$$

Rješenje. Imamo

$$f'(x) = \frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right)$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(f)

Derivirajte funkciju

$$f(x) := \operatorname{arctg} x + \operatorname{arcctg} x.$$

Rješenje. Imamo

$$f'(x) = \frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right)$$
$$= 0.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 21(f)

Derivirajte funkciju

$$f(x) := \operatorname{arctg} x + \operatorname{arcctg} x.$$

Rješenje. Imamo

$$f'(x) = \frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right) = 0.$$

Napomena. Zapravo je

$$\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}, \qquad x \in \mathbb{R}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Pravilo 2

Za funkcije $f, g : D \subseteq \mathbb{R} \to \mathbb{R}$ vrijedi:

- $(f \cdot g)' = f' \cdot g + f \cdot g'$
- $\left(\dfrac{f}{g}\right)' = \dfrac{f' \cdot g - f \cdot g'}{g^2}$,

kad god je desna strana definirana.

Zadatak 22(a)

Derivirajte funkciju

$$f(x) := 2^x \cdot \arcsin x.$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 22(a)

Derivirajte funkciju

$$f(x) := 2^x \cdot \arcsin x.$$

Rješenje. Imamo

$$f'(x) = (2^x)' \cdot \arcsin x + 2^x \cdot (\arcsin x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 22(a)

Derivirajte funkciju

$$f(x) := 2^x \cdot \arcsin x.$$

Rješenje. Imamo

$$f'(x) = (2^x)' \cdot \arcsin x + 2^x \cdot (\arcsin x)'$$
$$= 2^x \ln 2 \cdot \arcsin x + 2^x \cdot \frac{1}{\sqrt{1-x^2}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 22(b)

Derivirajte funkciju

$$f(x) := \frac{2x+3}{x^2-5x+5}.$$

Derivirajte funkciju

$$f(x) := \frac{2x+3}{x^2-5x+5}.$$

Rješenje. Imamo

$$f'(x) = \frac{(2x+3)' \cdot \left(x^2-5x+5\right) - (2x+3) \cdot \left(x^2-5x+5\right)'}{\left(x^2-5x+5\right)^2}$$

Zadatak 22(b)

Derivirajte funkciju

$$f(x) := \frac{2x+3}{x^2-5x+5}.$$

Rješenje. Imamo

$$f'(x) = \frac{(2x+3)' \cdot \left(x^2-5x+5\right) - (2x+3) \cdot \left(x^2-5x+5\right)'}{\left(x^2-5x+5\right)^2}$$
$$= \frac{2\left(x^2-5x+5\right) - (2x+3)(2x-5)}{\left(x^2-5x+5\right)^2}$$

Zadatak 22(b)

Derivirajte funkciju

$$f(x) := \frac{2x+3}{x^2-5x+5}.$$

Rješenje. Imamo

$$\begin{aligned} f'(x) &= \frac{(2x+3)' \cdot \left(x^2-5x+5\right) - (2x+3) \cdot \left(x^2-5x+5\right)'}{\left(x^2-5x+5\right)^2} \\ &= \frac{2\left(x^2-5x+5\right) - (2x+3)(2x-5)}{\left(x^2-5x+5\right)^2} \\ &= \frac{-2x^2-6x+25}{\left(x^2-5x+5\right)^2}. \end{aligned}$$

Zadatak 22(c)

Derivirajte funkciju

$$f(x) := \frac{\sin x + \cos x}{\sin x - \cos x}.$$

Zadatak 22(c)

Derivirajte funkciju

$$f(x) := \frac{\sin x + \cos x}{\sin x - \cos x}.$$

$$f'(x) = \frac{(\sin x + \cos x)'(\sin x - \cos x) - (\sin x + \cos x)(\sin x - \cos x)'}{(\sin x - \cos x)^2}$$

Zadatak 22(c)

Derivirajte funkciju

$$f(x) := \frac{\sin x + \cos x}{\sin x - \cos x}.$$

Rješenje. Koristeći da je

$$(\cos x)' = -\sin x \qquad \text{i} \qquad (\sin x)' = \cos x,$$

imamo

$$f'(x) = \frac{(\sin x + \cos x)'(\sin x - \cos x) - (\sin x + \cos x)(\sin x - \cos x)'}{(\sin x - \cos x)^2}$$

Zadatak 22(c)

Derivirajte funkciju

$$f(x) := \frac{\sin x + \cos x}{\sin x - \cos x}.$$

Rješenje. Koristeći da je

$$(\cos x)' = -\sin x \qquad \text{i} \qquad (\sin x)' = \cos x,$$

imamo

$$\begin{aligned} f'(x) &= \frac{(\sin x + \cos x)'(\sin x - \cos x) - (\sin x + \cos x)(\sin x - \cos x)'}{(\sin x - \cos x)^2} \\ &= \frac{(\cos x - \sin x)(\sin x - \cos x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2} \end{aligned}$$

Zadatak 22(c)

Derivirajte funkciju

$$f(x) := \frac{\sin x + \cos x}{\sin x - \cos x}.$$

Rješenje. Koristeći da je

$$(\cos x)' = -\sin x \qquad \text{i} \qquad (\sin x)' = \cos x,$$

imamo

$$\begin{aligned} f'(x) &= \frac{(\sin x + \cos x)'(\sin x - \cos x) - (\sin x + \cos x)(\sin x - \cos x)'}{(\sin x - \cos x)^2} \\ &= \frac{(\cos x - \sin x)(\sin x - \cos x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2} \\ &\overset{\text{sami}}{=} -\frac{2}{(\sin x - \cos x)^2}. \end{aligned}$$

Zadatak 22(d)

Derivirajte funkciju

$$f(x) := \frac{x^2}{\ln x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

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Zadatak 22(d)

Derivirajte funkciju

$$f(x) := \frac{x^2}{\ln x}.$$

Rješenje. Imamo

$$f'(x) = \frac{(x^2)' \cdot \ln x - x^2 \cdot (\ln x)'}{\ln^2 x}$$

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Zadatak 22(d)

Derivirajte funkciju

$$f(x) := \frac{x^2}{\ln x}.$$

Rješenje. Imamo

$$f'(x) = \frac{(x^2)' \cdot \ln x - x^2 \cdot (\ln x)'}{\ln^2 x} = \frac{2x \cdot \ln x - x^2 \cdot \frac{1}{x}}{\ln^2 x}$$

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Zadatak 22(d)

Derivirajte funkciju

$$f(x) := \frac{x^2}{\ln x}.$$

Rješenje. Imamo

$$f'(x) = \frac{(x^2)' \cdot \ln x - x^2 \cdot (\ln x)'}{\ln^2 x} = \frac{2x \cdot \ln x - x^2 \cdot \frac{1}{x}}{\ln^2 x} = \frac{2x \ln x - x}{\ln^2 x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Kompozicija funkcija

Definicija. **Kompozicija funkcija** $f : A \to B$ i $g : C \to D$ je funkcija $g \circ f : A \to D$,

$$(g \circ f)(x) := g(f(x)).$$

(Da bi bila definirana, za svaki $x \in A$ mora biti $f(x) \in C$.)

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Primjer. (a) $h(x) := (2x + 1)^3$

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$$\rightsquigarrow h = g \circ f \text{ za } f(x) := 2x + 1 \text{ i } g(t) := t^3.$$

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(b) $h(x) := 2^{\operatorname{tg} x}$

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$\rightsquigarrow h = g \circ f$ za $f(x) := 2x + 1$ i $g(t) := t^3$.

(b) $h(x) := 2^{\operatorname{tg} x}$

$\rightsquigarrow h = g \circ f$ za $f(x) := \operatorname{tg} x$ i $g(t) := 2^t$.

Pravilo 3

Vrijedi

$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x),$$

kad god je desna strana definirana.

Pravilo 3

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Primjer. Derivirajmo funkciju

$$h(x) := (2x + 1)^{10}.$$

Pravilo 3

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Primjer. Derivirajmo funkciju

$$h(x) := (2x + 1)^{10}.$$

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$$f(x) := 2x + 1 \qquad \text{i} \qquad g(t) := t^{10}.$$

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$$(g \circ f)'(x) = g'(f(x)) \cdot f'(x),$$

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$$h(x) := (2x + 1)^{10}.$$

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$$f(x) := 2x + 1 \qquad \text{i} \qquad g(t) := t^{10}.$$

Kako je

$$f'(x) = 2 \qquad \text{i} \qquad g'(t) = 10t^9,$$

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$$f'(x) = 2 \qquad \text{i} \qquad g'(t) = 10t^9,$$

imamo

$$h'(x) = g'(f(x)) \cdot f'(x) = 10(2x+1)^9 \cdot 2 = 20(2x+1)^9.$$

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kad god je desna strana definirana.

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$$f(x) := 2x + 1 \qquad \text{i} \qquad g(t) := t^{10}.$$

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imamo

$$h'(x) = g'(f(x)) \cdot f'(x) = 10(2x + 1)^9 \cdot 2 = 20(2x + 1)^9.$$

Zadatak 23(a)

Derivirajte funkciju

$$h(x) := \frac{1}{2x-1}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 23(a)

Derivirajte funkciju

$$h(x) := \frac{1}{2x-1}.$$

Rješenje. Imamo

$$h'(x) = -\frac{1}{(2x-1)^2} \cdot (2x-1)'$$

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Zadatak 23(a)

Derivirajte funkciju

$$h(x) := \frac{1}{2x-1}.$$

Rješenje. Imamo

$$h'(x) = -\frac{1}{(2x-1)^2} \cdot (2x-1)'$$
$$= -\frac{1}{(2x-1)^2} \cdot 2$$

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Zadatak 23(a)

Derivirajte funkciju

$$h(x) := \frac{1}{2x - 1}.$$

Rješenje. Imamo

$$h'(x) = -\frac{1}{(2x-1)^2} \cdot (2x-1)' = -\frac{1}{(2x-1)^2} \cdot 2 = -\frac{2}{(2x-1)^2}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(a^x)' = a^x \ln a \quad (a > 0)$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

Rješenje. Imamo

$$h'(x) = e^{\sin^2 x} \cdot \left(\sin^2 x\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

Rješenje. Imamo

$$h'(x) = e^{\sin^2 x} \cdot \left(\sin^2 x\right)'$$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

Rješenje. Imamo

$$h'(x) = e^{\sin^2 x} \cdot \left(\sin^2 x\right)' = e^{\sin^2 x} \cdot 2 \sin x \cdot (\sin x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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Zadatak 23(b)

Derivirajte funkciju

$$h(x) := e^{\sin^2 x}.$$

Rješenje. Imamo

$$\begin{aligned} h'(x) &= e^{\sin^2 x} \cdot (\sin^2 x)' \\ &= e^{\sin^2 x} \cdot 2\sin x \cdot (\sin x)' \\ &= e^{\sin^2 x} \cdot 2\sin x \cdot \cos x. \end{aligned}$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(c)

Derivirajte funkciju

$$h(x) := \sqrt{\operatorname{ctg} x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(c)

Derivirajte funkciju

$$h(x) := \sqrt{\operatorname{ctg} x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(c)

Derivirajte funkciju

$$h(x) := \sqrt{\operatorname{ctg} x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{\operatorname{ctg} x}} \cdot (\operatorname{ctg} x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(c)

Derivirajte funkciju

$$h(x) := \sqrt{\operatorname{ctg} x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{\operatorname{ctg} x}} \cdot (\operatorname{ctg} x)'$$
$$= \frac{1}{2\sqrt{\operatorname{ctg} x}} \cdot \left(-\frac{1}{\sin^2 x}\right).$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(d)

Derivirajte funkciju

$$h(x) := \sqrt{1 + \arcsin x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(d)

Derivirajte funkciju

$$h(x) := \sqrt{1 + \arcsin x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(d)

Derivirajte funkciju

$$h(x) := \sqrt{1 + \arcsin x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{1+\arcsin x}} \cdot (1 + \arcsin x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(d)

Derivirajte funkciju

$$h(x) := \sqrt{1 + \arcsin x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{1 + \arcsin x}} \cdot (1 + \arcsin x)'$$
$$= \frac{1}{2\sqrt{1 + \arcsin x}} \cdot \frac{1}{\sqrt{1 - x^2}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(e)

Derivirajte funkciju

$$h(x) := \sqrt{xe^x + x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(e)

Derivirajte funkciju

$$h(x) := \sqrt{xe^x + x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

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$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

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$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(e)

Derivirajte funkciju

$$h(x) := \sqrt{xe^x + x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{xe^x + x}} \cdot (xe^x + x)'$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(e)

Derivirajte funkciju

$$h(x) := \sqrt{xe^x + x}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{2\sqrt{xe^x + x}} \cdot (xe^x + x)'$$
$$= \frac{1}{2\sqrt{xe^x + x}} \left(x'e^x + x\,(e^x)' + 1\right)$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(e)

Derivirajte funkciju

$$h(x) := \sqrt{xe^x + x}.$$

Rješenje. Imamo

$$\begin{aligned} h'(x) &= \frac{1}{2\sqrt{xe^x + x}} \cdot (xe^x + x)' \\ &= \frac{1}{2\sqrt{xe^x + x}} \left(x'e^x + x\,(e^x)' + 1\right) \\ &= \frac{1}{2\sqrt{xe^x + x}} \left(e^x + xe^x + 1\right). \end{aligned}$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$h'(x) = \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)'$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$h'(x) = \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)'$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

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$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

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$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$h'(x) = \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)'$$
$$= \frac{1}{3} \cdot (2e^x - 2^x + 1)^{-\frac{2}{3}} \cdot (2e^x - 2^x + 1)'$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

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$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$h'(x) = \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)'$$
$$= \frac{1}{3} \cdot (2e^x - 2^x + 1)^{-\frac{2}{3}} \cdot (2e^x - 2^x + 1)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Deriviraj te funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$h'(x) = \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)'$$
$$= \frac{1}{3} \cdot (2e^x - 2^x + 1)^{-\frac{2}{3}} \cdot (2e^x - 2^x + 1)'$$
$$+ 5\,(\ln x)^4 \cdot (\ln x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(f)

Derivirajte funkciju

$$h(x) := \sqrt[3]{2e^x - 2^x + 1} + (\ln x)^5.$$

Rješenje. Imamo

$$\begin{aligned} h'(x) &= \left(\sqrt[3]{2e^x - 2^x + 1}\right)' + \left((\ln x)^5\right)' \\ &= \frac{1}{3} \cdot (2e^x - 2^x + 1)^{-\frac{2}{3}} \cdot (2e^x - 2^x + 1)' \\ &\quad + 5\,(\ln x)^4 \cdot (\ln x)' \\ &= \frac{1}{3} \cdot (2e^x - 2^x + 1)^{-\frac{2}{3}} \cdot (2e^x - 2^x \ln 2) \\ &\quad + 5\,(\ln x)^4 \cdot \frac{1}{x}. \end{aligned}$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(g)

Derivirajte funkciju

$$h(x) := \arcsin \frac{1}{x^2}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(g)

Derivirajte funkciju

$$h(x) := \arcsin \frac{1}{x^2}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(g)

Derivirajte funkciju

$$h(x) := \arcsin \frac{1}{x^2}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{x^2}\right)^2}} \cdot \left(\frac{1}{x^2}\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(g)

Derivirajte funkciju

$$h(x) := \arcsin \frac{1}{x^2}.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\sqrt{1 - \left(\frac{1}{x^2}\right)^2}} \cdot \left(\frac{1}{x^2}\right)'$$

$$= \frac{1}{\sqrt{1 - \frac{1}{x^4}}} \cdot \left(-\frac{2}{x^3}\right).$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(h)

Derivirajte funkciju

$$h(x) := \frac{1}{5^{x^2}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(h)

Derivirajte funkciju

$$h(x) := \frac{1}{5^{x^2}}.$$

Rješenje. Imamo

$$h'(x) = \left(5^{-x^2}\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(h)

Derivirajte funkciju

$$h(x) := \frac{1}{5^{x^2}}.$$

Rješenje. Imamo

$$h'(x) = \left(5^{-x^2}\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(h)

Derivirajte funkciju

$$h(x) := \frac{1}{5^{x^2}}.$$

Rješenje. Imamo

$$h'(x) = \left(5^{-x^2}\right)'$$
$$= 5^{-x^2} \ln 5 \cdot \left(-x^2\right)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(h)

Derivirajte funkciju

$$h(x) := \frac{1}{5^{x^2}}.$$

Rješenje. Imamo

$$h'(x) = \left(5^{-x^2}\right)' = 5^{-x^2}\ln 5 \cdot \left(-x^2\right)' = 5^{-x^2}\ln 5 \cdot (-2x)\,.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(i)

Derivirajte funkciju

$$h(x) := \log \sin x.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(i)

Derivirajte funkciju

$$h(x) := \log \sin x.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(i)

Derivirajte funkciju

$$h(x) := \log \sin x.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\sin x \cdot \ln 10} \cdot (\sin x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(i)

Derivirajte funkciju

$$h(x) := \log \sin x.$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\sin x \cdot \ln 10} \cdot (\sin x)' = \frac{\cos x}{\sin x \cdot \ln 10}$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(i)

Derivirajte funkciju

$$h(x) := \log \sin x.$$

Rješenje. Imamo

$$\begin{aligned} h'(x) &= \frac{1}{\sin x \cdot \ln 10} \cdot (\sin x)' \\ &= \frac{\cos x}{\sin x \cdot \ln 10} \\ &= \frac{\operatorname{ctg} x}{\ln 10}. \end{aligned}$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(j)

Derivirajte funkciju

$$h(x) := \sqrt{e^{2x}}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

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Zadatak 23(j)

Derivirajte funkciju

$$h(x) := \sqrt{e^{2x}}.$$

Rješenje. Primijetimo da je

$$h(x) = \left(e^{2x}\right)^{\frac{1}{2}}$$

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Zadatak 23(j)

Derivirajte funkciju

$$h(x) := \sqrt{e^{2x}}.$$

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Zadatak 23(j)

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Zadatak 23(j)

Derivirajte funkciju

$$h(x) := \sqrt{e^{2x}}.$$

Rješenje. Primijetimo da je

$$h(x) = \left(e^{2x}\right)^{\frac{1}{2}} = e^{2x\cdot\frac{1}{2}} = e^{x}$$

pa je

$$h'(x) = e^{x}.$$

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Zadatak 23(k)

Derivirajte funkciju

$$h(x) := \ln\ln\left(3 - 2x^2\right).$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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Zadatak 23(k)

Derivirajte funkciju

$$h(x) := \ln \ln \left(3 - 2x^2\right).$$

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Zadatak 23(k)

Derivirajte funkciju

$$h(x) := \ln \ln \left(3 - 2x^2\right).$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\ln\left(3 - 2x^2\right)} \cdot \left(\ln\left(3 - 2x^2\right)\right)'$$

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Zadatak 23(k)

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Zadatak 23(k)

Derivirajte funkciju

$$h(x) := \ln \ln\left(3 - 2x^2\right).$$

Rješenje. Imamo

$$h'(x) = \frac{1}{\ln(3-2x^2)} \cdot \left(\ln\left(3 - 2x^2\right)\right)'$$

$$= \frac{1}{\ln(3-2x^2)} \cdot \frac{1}{3-2x^2} \cdot \left(3 - 2x^2\right)'$$

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Zadatak 23(k)

Derivirajte funkciju

$$h(x) := \ln \ln\left(3 - 2x^2\right).$$

Rješenje. Imamo

$$\begin{aligned} h'(x) &= \frac{1}{\ln\left(3-2x^2\right)} \cdot \left(\ln\left(3-2x^2\right)\right)' \\ &= \frac{1}{\ln\left(3-2x^2\right)} \cdot \frac{1}{3-2x^2} \cdot \left(3-2x^2\right)' \\ &= \frac{1}{\ln\left(3-2x^2\right)} \cdot \frac{1}{3-2x^2} \cdot (-4x). \end{aligned}$$

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Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

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Zadatak 23(I)

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Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1-\arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

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Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

Rješenje. Imamo

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Zadatak 23(I)

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$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

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Zadatak 23(I)

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$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

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$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

$$+ \frac{1}{2} \cdot 2(1 - \arccos(3x)) \cdot (1 - \arccos(3x))'$$

$$= 2^{\arcsin(3x)} \ln 2 \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

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$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

$(\ln x)' = \frac{1}{x} \quad (x > 0)$

Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1-\arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

$$+ \frac{1}{2} \cdot 2(1-\arccos(3x)) \cdot (1-\arccos(3x))'$$

$$= 2^{\arcsin(3x)} \ln 2 \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)'$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1-\arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$

$$+ \frac{1}{2} \cdot 2(1-\arccos(3x)) \cdot (1-\arccos(3x))'$$

$$= 2^{\arcsin(3x)} \ln 2 \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot (3x)'$$

$$+ (1-\arccos(3x)) \left(- \left(-\frac{1}{\sqrt{1-(3x)^2}} \right) \right) \cdot (3x)'$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 23(I)

Derivirajte funkciju

$$h(x) := 2^{\arcsin(3x)} + \frac{(1 - \arccos(3x))^2}{2}.$$

Rješenje. Imamo

$$h'(x) = 2^{\arcsin(3x)} \ln 2 \cdot (\arcsin(3x))'$$
$$+ \frac{1}{2} \cdot 2(1 - \arccos(3x)) \cdot (1 - \arccos(3x))'$$
$$= 2^{\arcsin(3x)} \ln 2 \cdot \frac{1}{\sqrt{1 - (3x)^2}} \cdot (3x)'$$
$$+ (1 - \arccos(3x)) \left(- \left(- \frac{1}{\sqrt{1 - (3x)^2}} \right) \right) \cdot (3x)'$$
$$= 2^{\arcsin(3x)} \ln 2 \cdot \frac{1}{\sqrt{1 - (3x)^2}} \cdot 3$$
$$+ (1 - \arccos(3x)) \cdot \frac{1}{\sqrt{1 - (3x)^2}} \cdot 3.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln x}\right)^x = e^{x \cdot \ln x},$$

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Zadatak 24(a)

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln x}\right)^x = e^{x \cdot \ln x},$$

imamo

$$h'(x) = e^{x \cdot \ln x} \cdot (x \cdot \ln x)'$$

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln x}\right)^x = e^{x \cdot \ln x},$$

imamo

$$\begin{aligned} h'(x) &= e^{x \cdot \ln x} \cdot (x \cdot \ln x)' \\ &= e^{x \cdot \ln x} \cdot (x' \cdot \ln x + x \cdot (\ln x)') \end{aligned}$$

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln x}\right)^x = e^{x \cdot \ln x},$$

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$$\begin{aligned} h'(x) &= e^{x \cdot \ln x} \cdot (x \cdot \ln x)' \\ &= e^{x \cdot \ln x} \cdot (x' \cdot \ln x + x \cdot (\ln x)') \\ &= e^{x \cdot \ln x} \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) \end{aligned}$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

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Zadatak 24(a)

Derivirajte funkciju

$$h(x) := x^x.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln x}\right)^x = e^{x \cdot \ln x},$$

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$$\begin{aligned} h'(x) &= e^{x \cdot \ln x} \cdot (x \cdot \ln x)' \\ &= e^{x \cdot \ln x} \cdot (x' \cdot \ln x + x \cdot (\ln x)') \\ &= e^{x \cdot \ln x} \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) \\ &= x^x \cdot (\ln x + 1). \end{aligned}$$

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Zadatak 24(b)

Derivirajte funkciju

$$h(x) := (\sin x)^{\cos x}.$$

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 24(b)

Derivirajte funkciju

$$h(x) := (\sin x)^{\cos x}.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln \sin x}\right)^{\cos x} = e^{\cos x \cdot \ln \sin x},$$

imamo

$c' = 0 \quad (c \in \mathbb{R} \text{ konstanta})$

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$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

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Zadatak 24(b)

Derivirajte funkciju

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Zadatak 24(b)

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Rješenje. Kako je

$$h(x) = \left(e^{\ln \sin x}\right)^{\cos x} = e^{\cos x \cdot \ln \sin x},$$

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$$h'(x) = e^{\cos x \cdot \ln \sin x} \cdot (\cos x \cdot \ln \sin x)'$$

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Derivirajte funkciju

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$$h(x) = \left(e^{\ln \sin x}\right)^{\cos x} = e^{\cos x \cdot \ln \sin x},$$

imamo

$$h'(x) = e^{\cos x \cdot \ln \sin x} \cdot (\cos x \cdot \ln \sin x)'$$

$$= (\sin x)^{\cos x} \left((\cos x)' \cdot \ln \sin x + \cos x \cdot (\ln \sin x)'\right)$$

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Zadatak 24(b)

Derivirajte funkciju

$$h(x) := (\sin x)^{\cos x}.$$

Rješenje. Kako je

$$h(x) = \left(e^{\ln \sin x}\right)^{\cos x} = e^{\cos x \cdot \ln \sin x},$$

imamo

$$h'(x) = e^{\cos x \cdot \ln \sin x} \cdot (\cos x \cdot \ln \sin x)'$$
$$= (\sin x)^{\cos x} \left((\cos x)' \cdot \ln \sin x + \cos x \cdot (\ln \sin x)'\right)$$

$c' = 0 \quad (c \in \mathbb{R}$ konstanta$)$

$x' = 1$

$(x^n)' = nx^{n-1} \quad (n \in \mathbb{Z})$

$(a^x)' = a^x \ln a \quad (a > 0)$

$(e^x)' = e^x$

$(x^a)' = ax^{a-1} \quad (a \in \mathbb{R},\ x > 0)$

$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \quad (x > 0)$

$(\sin x)' = \cos x$

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$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

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$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \quad (|x| < 1)$

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$(\log_a x)' = \frac{1}{x \ln a} \quad (a > 0, a \neq 1)$

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Derivacije višeg reda

$$
\begin{aligned}
f'' &:= f^{(2)} := \left(f'\right)' \\
f''' &:= f^{(3)} := \left(f''\right)' \\
&\; f^{(4)} := \left(f'''\right)' \\
&\; f^{(5)} := \left(f^{(4)}\right)' \\
&\quad\vdots \\
&\; f^{(n+1)} := \left(f^{(n)}\right)' \\
&\quad\vdots
\end{aligned}
$$

Zadatak 25

Zadana je funkcija

$$f(x) := \ln(1 + x).$$

Izračunajte $f'''(0)$.

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Dakle,

$$f'''(0) = \frac{2}{(1+0)^3} = 2.$$